Exercise 68

Suppose f is differentiable on $\mathbb{R}$ and α is a real number. Let $F(x) = f(x^\alpha)$ and $G(x) = [f(x)]^\alpha$. Find expressions for (a) $F'(x)$ and (b) $G'(x)$.

Solution

Take the derivative of $F(x)$.

$$\begin{aligned} F'(x) &= \frac{d}{dx}[f(x^\alpha)] \\ &= f'(x^\alpha) \cdot \frac{d}{dx}(x^\alpha) \\ &= f'(x^\alpha) \cdot (\alpha x^{\alpha-1}) \end{aligned}$$

Take the derivative of $G(x)$.

$$\begin{aligned} G'(x) &= \frac{d}{dx}\{[f(x)]^\alpha\} \\ &= \alpha[f(x)]^{\alpha-1} \cdot \frac{d}{dx}[f(x)] \\ &= \alpha[f(x)]^{\alpha-1} \cdot f'(x) \end{aligned}$$